Flavor Singlet Axial Coupling of the Proton – An Updated Analysis *

J. Viehoff[a] for the SESAM[a,b] Collaboration

[a]NIC c/o Forschungszentrum Jülich, D-52425 Jülich, and DESY, D-22603 Hamburg, Germany.
[b]Physics Department, University of Wuppertal, D-42097 Wuppertal, Germany.

We present a combined analysis of SESAM and $T\chi L$ data for the flavor singlet axial coupling G_A^1 of the proton, which is very helpful to stabilize the disconnected signals at small quark masses. From connected and disconnected contributions we use the tadpole improved renormalization constant Z_A and obtain $G_A^1 = 0.21(12)$.

1. Introduction

The flavor singlet axial coupling of the proton is defined as

$$s_\mu G_A^1 = \langle P|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|P\rangle, \quad (1)$$

where s_μ denotes the components of the proton polarization vector. In the naive parton model G_A^1 is related to the fraction of the proton spin carried by the quarks. From the measurement of the first moment of the spin dependent proton structure function, g_p^1, in deep inelastic polarized muon proton scattering the EMC experiment [1] found a small value,

$$G_A^1 = 0.12(17) \quad (2)$$

which led to the “proton spin crisis”. The result for G_A^1 indicates that the contribution to the proton spin from the quarks is small. New experiments, including proton, neutron and deuteron data [2], have shifted the value up to $G_A^1 = 0.29(6)$ which is still far away from the Ellis-Jaffe QCD sum-rule prediction [3]:

$$G_A^1 \simeq G_A^8 = 0.579(25) \quad (3)$$

Albeit a lot of theoretical and experimental work has been done in the meantime, a clear understanding of the proton spin is still lacking. In this context lattice calculations are very illuminating though hampered by large fluctuations [4].

2. Lattice techniques

On the lattice both connected and disconnected diagrams for G_A^1 (see figure 1) can be calculated from ratios of correlation functions [4].

*Poster presented by J. Viehoff.

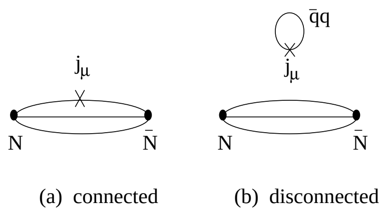

Figure 1. *Connected (a) and disconnected (b) diagrams contribute to the flavor singlet axial coupling G_A^1 of the proton ($j_\mu = \bar{q}\gamma_\mu\gamma_5 q$).*

The SESAM collaboration has analyzed 200 $16^3 \times 32$ gauge configurations at each of their four quark masses corresponding to $m_\pi/m_\rho = 0.833...0.686$. The configurations have been generated previously in the standard Wilson discretization scheme with $n_f = 2$ mass degenerate quark flavors, at $\beta = 5.6$. In addition 200 $T\chi L$ gauge configurations ($24^3 \times 40$ lattices) at the same coupling $\beta = 5.6$ have been analyzed. The quark mass of the $T\chi L$ lattices corresponds to the lightest SESAM quark mass. Details of the simulations can be found in [5,6].

2.1. Connected contributions

For the amplitudes from the connected diagrams (see figure 1(a)) in the matrix element, eq. 1, we have applied the global summation method and the insertion technique as described in [4].

From the ratio

$$R_{A_\mu}^{SUM} = \frac{\sum_{\vec{x}}\langle P^\dagger \sum_{\vec{y},y_0} [\bar{q}\gamma_\mu\gamma_5 q]\,(\vec{y},y_0)P\rangle}{\sum_{\vec{x}}\langle P^\dagger P\rangle}$$

$$- \quad \langle \sum_{\vec{y},y_0} [\bar{q}\gamma_\mu\gamma_5 q]\,(\vec{y},y_0)\rangle \;, \tag{4}$$

where P is an interpolating operator for the proton, the connected amplitude $C_q = \langle P|\bar{q}\gamma_\mu\gamma_5 q|P\rangle_{conn}$ can be obtained from the asymptotic linear slope

$$R^{SUM}_{A_\mu}(t) \overset{t\to\infty}{\to} A + \langle P|\bar{q}\gamma_\mu\gamma_5 q|P\rangle\, t \;. \tag{5}$$

2.2. Disconnected contributions

For the disconnected amplitudes $D_q = \langle P|\bar{q}\gamma_\mu\gamma_5 q|P\rangle_{disc}$ (see figure 1(b)) we have used the plateau accumulation method (PAM) [4]: from the partial summation

$$R^{PAM}_{A_\mu}(t,\Delta t_0,\Delta t) = \sum_{y_0=\Delta t_0}^{t-\Delta t} R^{PLA}_{A_\mu}(t,y_0) \;, \tag{6}$$

with

$$R^{PLA}_{A_\mu} = \frac{\sum_{\vec{x}}\langle P^\dagger \sum_{\vec{y}} [\bar{q}\gamma_\mu\gamma_5 q]\,(\vec{y},y_0) P\rangle}{\sum_{\vec{x}}\langle P^\dagger P\rangle} - \langle \sum_{\vec{y}} [\bar{q}\gamma_\mu\gamma_5 q]\,(\vec{y},y_0)\rangle \;. \tag{7}$$

D_q follows from the asymptotic time dependence ($t\to\infty$) in

$$R^{PAM}_{A_\mu}(t,\Delta t_0,\Delta t) = B + D_q(t-\Delta t-\Delta t_0) \;. \tag{8}$$

For the calculation of the axial vector quark loops we have used the spin explicit stochastic estimator technique with complex Z2 noise and 100 stochastic estimates per spin component and gauge field configuration [4].

3. Raw data

Figure 2 displays $R^{SUM}_A(t)$ for the connected amplitudes $C_{u,d}$ on the four SESAM quark masses. The results from the linear fits are summarized in table 1. For the lightest SESAM quark mass ($\kappa_{sea} = 0.1575$) we have also calculated $C_{u,d}$ on the $T\chi L$ lattice. Note that the volume effects are less than 7%.

In Figure 3 we plotted $R^{PAM}_A(t)$ ($\Delta t = \Delta t_0 = 1$) for the disconnected amplitudes with $\kappa_{sea} = 0.156$ (SESAM) and $\kappa_{sea} = 0.1575$ ($T\chi L$). Albeit the statistical errors for D_q are large we observe comparable quality of signals from both plots.

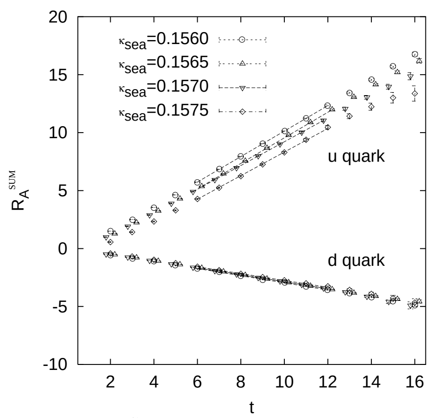

Figure 2. *$R^{SUM}_A(t)$ for the SESAM gauge-configurations (spin-averaged). The linear fits give the connected amplitudes C_u and C_d.*

κ	C_u	C_d
0.1560	1.100(8)	-0.307(3)
0.1565	1.102(11)	-0.308(4)
0.1570	1.025(12)	-0.297(8)
0.1575(SESAM)	1.018(24)	-0.295(10)
0.1575($T\chi L$)	1.086(11)	-0.310(6)

Table 1
Connected amplitude $C_{u,d}$ from the linear fits to $R^{SUM}_A(t)$ (SESAM and $T\chi L$).

4. Results

The chiral extrapolations of $C_{u,d}$ and $D_{q,s}$ to the light quark mass m_{light} are shown in figure 4. Since the finite volume effects are small we have taken the $T\chi L$ data at $\kappa_{sea} = 0.1575$ for the extrapolation in D_q. Within the errors the flavor symmetry $D_q = D_s$ in the disconnected amplitudes is preserved and we use only the connected amplitudes for G^8_A and G^3_A.

With the Wilson discretization both the flavor singlet and non-singlet current need renormalization. We have used the first order (tadpole improved) result from lattice perturbation theory for Z^S_A and Z^{NS}_A[4]. The renormalized coupling constants of the proton and the contributions Δq to the proton spin are listed in table 2.

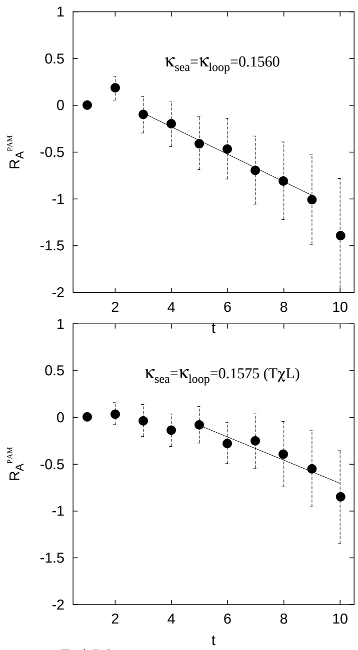

Figure 3. $R_A^{PAM}(t)$ *for* $\kappa_{sea} = 0.156$ *(SESAM) and* $\kappa_{sea} = 0.1575$ *($T\chi L$) (spin-averaged). The linear fits give the disconnected amplitude* D_q.

Δu	Δd	Δs
0.62(7)	-0.28(6)	-0.12(7)
G_A^1	G_A^3	G_A^8
0.21(12)	0.907(20)	0.484(18)

Table 2
Renormalized coupling constants of the proton and the contributions Δq *to the proton spin.*

5. Conclusion

We have calculated connected and disconnected contributions to the flavor singlet axial vector coupling of the proton in a full QCD $n_f = 2$ lattice simulation with Wilson fermions. We have checked the volume effects for the connected amplitudes when we increase the lattice size at the same coupling ($\beta = 5.6$) and quark mass. In the analysis of the disconnected contributions we have replaced the SESAM results for the lightest quark mass by the $T\chi L$ data and obtained a better signal for D_q. Nevertheless, the errors are dominated by the large statistical fluctuations in the disconnected amplitudes. We

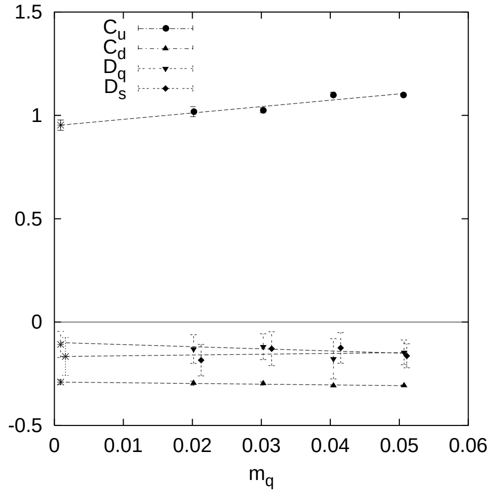

Figure 4. *Linear chiral extrapolations of* $C_{u,d}$ *and* $D_{q,s}$ *to the light quark mass* m_{light}.

find $G_A^1 = 0.21(12)$ to be consistent with the result from experiment and with previous quenched estimates [7,8]. For the triplet coupling we get $G_A^3 = 0.907(20)$ which is 30% smaller than the experimental value, $G_A^3 = 1.2670(35)$ [2]. To fix the systematic errors in the lattice calculation we would need a scaling analysis as well as a non-perturbative determination of Z_A^{NS} and Z_A^S. q

REFERENCES

1. EMC, J. Ashman et al., Nucl. Phys. **B328** (1989) 1; Phys. Lett. **B206** (1988)364.
2. SMC, D. Adams et al., Phys. Rev. **D56** (1997)5330.
3. J. Ellis, R.L. Jaffe, Phys. Rev. **D9** (1974) 1444; Phys. Rev. **D10** (1974)1669.
4. SESAM collaboration, hep-lat/9901009, appears in Phys.Rev. **D**.
5. SESAM/$T\chi L$ collaboration, Nucl.Phys. Proc.Suppl. **60A** (1998) 311.
6. SESAM Collaboration, Phys.Rev. **D59**, 014509 (1999).
7. M. Fukugita, Y. Kuramashi, M. Okawa, A. Ukawa, Phys. Rev. Lett. **75** (1995) 2092.
8. S.J. Dong, J.F. Lagaë, K.F. Liu, Phys. Rev. Lett. **75** (1995) 2096.